FACTORING TRINOMIALS OF THE FORM $x^2 + bx + c$, WHERE $c > 0$

- Before doing this exercise, you may want to study these:
  Basic Concepts Involved in Factoring Trinomials

(more mathematical cats)

Here, you will practice factoring trinomials of the form $x^2 + bx + c$, where b and c are integers, and $c > 0$.
That is, ***the constant term is positive***.

Recall that the ***integers*** are: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

As discussed in Basic Concepts Involved in Factoring Trinomials, you must first find two numbers that add to b and that multiply to c, since then:

$$x^2 + bx + c \;=\; x^2 + (\overbrace{f+g}^{=b})x + \overbrace{fg}^{=c} \;=\; (x+f)(x+g)$$

Since c is positive in this exercise, both numbers will be positive, or both numbers will be negative. (How can two numbers multiply to give a positive result? They must both be positive, or they must both be negative.) That is, both numbers will have the same sign.

When you add numbers that have the same sign, then ***in your head you actually do an addition problem***. For example, to mentally add $(-5) + (-3)$, in your head you would compute $5 + 3$, and then assign a negative sign to your answer.

The sign of b (the coefficient of the x term) determines the common sign of your numbers.
If $b > 0$, then both numbers will be positive.
If $b < 0$, then both numbers will be negative.

These results are summarized below:

FACTORING TRINOMIALS OF THE FORM $x^2 + bx + c$, $\quad c > 0$

- Check that the coefficient of the square term is 1.
- Check that the constant term (c) is positive.
- ***It's easier to do mental computations involving only positive numbers.*** So, if b (the coefficient of the x term) is negative, you'll initially ignore the minus sign.
  That is, in the next step, notice that you're working with the ***absolute value*** of b, which is positive.
- Find two numbers that ADD TO $|b|$ and MULTIPLY TO c.
- Now (and only now), you'll use the actual plus-or-minus sign of b.
  If $b > 0$, then both numbers will be positive.
  If $b < 0$, then both numbers will be negative.
- Use these two numbers to factor the trinomial, as illustrated in the examples below.
- Be sure to check your answer using FOIL.

EXAMPLES:

Question: Factor: $x^2 + 5x + 6$
Solution: Thought process:
Is the coefficient of the x^2 term equal to 1? Check!
Is the constant term positive? Check!
Find two numbers that add to 5 and multiply to 6.
The numbers 2 and 3 work, since $2 + 3 = 5$ and $2 \cdot 3 = 6$.
Since the coefficient of x is positive, both numbers will be positive.
The desired numbers are 2 and 3.
Then,

$$x^2 + 5x + 6 \;=\; x^2 + (\overbrace{2 + 3}^{=5})x + \overbrace{2 \cdot 3}^{=6} \;=\; (x + 2)(x + 3)$$

Check: $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

Question: Factor: $x^2 - 5x + 6$
Solution: Thought process:
Is the coefficient of the x^2 term equal to 1? Check!
Is the constant term positive? Check!
Find two numbers that add to 5 and multiply to 6.
(Notice that we initially 'throw away' the minus sign on the 5,
because it's easier to do mental arithmetic with positive numbers.)
The numbers 2 and 3 work, since $2 + 3 = 5$ and $2 \cdot 3 = 6$.
Since the coefficient of x is negative, both numbers will be negative.
(Here's where we now used the minus sign in the middle!)
The desired numbers are -2 and -3.
Then,

$$x^2 - 5x + 6 \;=\; x^2 + (\overbrace{(-2) + (-3)}^{=-5})x + \overbrace{(-2)(-3)}^{=6} \;=\; (x - 2)(x - 3)$$

Check: $(x - 2)(x - 3) = x^2 - 3x - 2x + 6 = x^2 - 5x + 6$

Question: Factor: $x^2 + x + 1$
Solution:
There are no integers that add to 1 and multiply to 1.
Thus, $x^2 + x + 1$ is not factorable over the integers.